Collective properties of indirect excitons in coupled quantum wells in random field

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Abstract

The influence of a random field induced by impurities, boundary irregularities etc. on the superfluidity of a quasi-two-dimensional ($2D$) system of spatially indirect excitons in coupled quantum wells is studied. The interaction between excitons is taken into account in the ladder approximation. The random field is allowed to be large compared to the dipole-dipole repulsion between excitons. The coherent potential approximation (CPA) allows us to derive the exciton Green's function for a wide range of the random field, and the CPA results are used in the weak-scattering limit, which results in the second-order Born approximation. The Green's function of the collective excitations for the cases of (1) equal electron and hole masses and (2) the "heavy hole" limit are derived analytically. For quasi-two-dimensional excitonic systems, the density of the superfluid component and the Kosterlitz-Thouless temperature of the superfluid phase transition are obtained, and are found to decrease as the random field increases. This puts constraints on the experimental efforts to observe excitonic superfluidity.

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I. INTRODUCTION

Superfluidity in a system of spatially indirect excitons in coupled quantum wells (CQW) has been predicted by Lozovik and Yudson,[1] and several subsequent theoretical studies[2,3,4,5,6,7,8,9,10,11,12,13,14,15] have suggested that this should be manifested as persistent electric currents, quasi-Josephson phenomena and unusual properties in strong magnetic fields. In the past ten years, a number of experimental studies have focused on observing these behaviors.[16,17,18,19,20,21,22,23] The coupled quantum well system is conceptually simple: negative electrons are trapped in a two-dimensional plane, while an equal number of positive holes is trapped in a parallel plane a distance D away. One of the appeals of this system is that the electron and hole wavefunctions have very little overlap, so that the excitons can have very long lifetime (> 100 ns), and therefore they can be treated as metastable particles to which quasiequlibrium statistics apply. Also when D is large enough, the interactions between the excitons are entirely dipole-dipole repulsive.

Much of the theory of excitonic systems has not taken into account the role of disorder, which is created by impurities and boundary irregularities of the quantum wells. In real experiments, however, disorder plays a very important role. Although the inhomogeneous broadening linewidth of typical GaAs-based samples has been improved from around 20 meV to less than 1 meV,[16] the disorder energy is still not much smaller than the exciton-exciton repulsion energy. At a typical exciton density of a few times 10^{10} cm^{-2}, the interaction energy of the excitons is several meV. On the other hand, the typical disorder energy of 1 meV is low compared to the typical exciton binding energy of 5 meV. Typical thermal energies at liquid helium temperatures are $k_BT = 0.2 - 2$ meV.

If the chemical potential of the interacting exciton system (controlled by the characteristic dipole-dipole interaction at fixed exciton density) is smaller than the characteristic disorder energy, the exciton system is expected to be localized in separate lakes in the minima of the random potential created by the disorder. The Bose condensation in this case is similar to that for Bose atoms in trap[24,25,26,27,28,29] (for the role of exciton tunneling, etc., see Ref. [30]). In the opposite limit, one has a translationally invariant, extended exciton system with a random potential field. In the latter case, the critical properties and quasi-long non-diagonal order are similar to the case without disorder (see, e.g., Ref. [31] and references cited therein), but the disorder suppresses the Kosterlitz-Thouless transition temperature

and the superfluid density.

Earlier studies of disorder in exciton systems included theory of the transport properties of direct and indirect excitons and magnetoexcitons in random fields,[32] the influence of various random fields on excitonic and magnetoexcitonic absorption of light,[33,34] and Anderson localization of excitons.[35] The effect of a weak random field on the collective properties and superfluidity of excitons in nonuniform systems was analyzed in Ref.[36], including the cases of a dilute gas of three-dimensional (3D) excitons, two-dimensional (2D) excitons in a single quantum well, and indirect excitons in coupled quantum wells in a random field. In that work the random field was assumed to be much smaller than the exciton-exciton interaction. In two-dimensional systems, the excitonic interaction in the Bogoliubov approximation is valid only in non-physically low densities because of the divergence of the two-dimensional scattering amplitude in the Born approximation.[37] Therefore, the ladder approximation must be used at low densities to treat properly the interaction between two-dimensional excitons.[37,38] In Ref.[36], the contribution to the exciton Green's function from the interaction of the excitons with the random field was derived by perturbation theory, which limited the strength of the random field that could be studied. In this paper we study the case of a random field which is not necessarily small compared to the dipole-dipole repulsion between excitons. The coherent potential approximation (CPA) allows us to derive the 2D indirect exciton Green's function for a wide range of random field strengths, resulting in the second order Born approximation in the weak scattering limit (the second order Born approximation Green's function for 3D direct excitons was obtained by Gevorkyan and Lozovik[35]). We predict that in the low-temperature limit, the density n_s of the superfluid component in CQW systems and the temperature of the superfluid transition (the Kosterlitz-Thouless temperature T_c[39]) are decreasing functions of the random field.

A typical example of a two-dimensional system of weakly interacting bosons is a system of indirect excitons in coupled quantum wells (GaAs/AlGaAs)[16,17,18,19,20,21,22]. Fluctuations of the thickness of a quantum well, which arise during the fabrication process, impurities in the system, and disorder in the alloy of the barriers can all lead to the appearance of a random field. Of these, spectral analysis of the exciton luminescence shows that alloy disorder, with a characteristic length scale short compared to the excitonic Bohr radius of around 100 Å, plays the most important role.

The paper is organized in the following way. In Sec. II the Green's function of the single

exciton in the random field is derived analytically in the coherent potential approximation (CPA) in the weak-scattering limit (resulting in the second order Born approximation) for the cases of (1) equal electron and hole masses and (2) the "heavy hole" limit. In Sec. III the Green's function and the the spectrum of collective excitations for a system of dilute indirect excitons in a random field are obtained within the ladder approximation. In Sec. IV the dependencies of the density of the superfluid component and the temperature of the Kosterlitz-Thouless transition on the random field are derived. In Sec. V we we present our conclusions and discuss possible experimental manifestations of superfluidity of indirect excitons in CQWs in a random field.

II. THE GREEN'S FUNCTION OF THE SINGLE EXCITON IN THE RANDOM FIELD

In our model, the random potential $V(\mathbf{r})$ acting on an electron and hole is considered to be delta function correlated Gaussian noise, such that

$$\langle V(\mathbf{r})V(\mathbf{r}')\rangle = g\delta^d(\mathbf{r}-\mathbf{r}'), \qquad \langle V(\mathbf{r})\rangle = 0, \tag{1}$$

where d is the dimensionality of the space (for the spatially indirect exciton in CQWs $d=2$). An electron is subjected to the potential $V_e = \alpha_e V(\mathbf{r})$ and a hole to $V_h = -\alpha_h V(\mathbf{r})$, where α_e and α_h are constants.

We consider the characteristic length of the random field potential l to be much lower than the average distance between excitons $r_s \sim 1/\sqrt{\pi n}$ ($l \ll 1/\sqrt{\pi n}$, where n is the total exciton density). Therefore, in order to obtain the Green's function of the excitons with dipole-dipole repulsion in the random field, we first obtain the Green's function of a single exciton in the random field (not interacting with other excitons), and then apply perturbation theory with respect to the dipole-dipole repulsion between excitons, using the system of the non-interacting excitons as a reference system.

The Green's function of the center of mass of the isolated exciton at $T=0$ in the momentum-frequency domain ($G^{(0)}(\mathbf{p},\omega)$) in the random field in the coherent potential approximation (CPA) is given by[35] (here and below $\hbar = 1$)

$$G^{(0)}(\mathbf{p},\omega) = \frac{1}{\omega - \varepsilon_0(p) + \mu + iQ(\mathbf{p},\omega)}, \tag{2}$$

where μ is the chemical potential of the system, and $\varepsilon_0(p) = p^2/2M$ is the spectrum of the center mass of the exciton in the "clean" system ($M = m_e + m_h$ is the mass of the exciton; m_e and m_h are the electron and hole masses, respectively). The function $Q(\mathbf{p},\omega)$ is determined by the effective random field acting on the center of mass of the exciton. For zero random field, $Q(\mathbf{p},\omega) \to 0$. If $gM \ll E_b$, where E_b is the binding energy of the indirect exciton ($E_b \sim e^2/\epsilon D$ at $D \gg \rho(D)$[40], D being the distance between e and h wells; $\rho(D) = (8a)^{1/4}D^{3/4}$ is the radius of the 2D indirect exciton at $D \gg a$, ϵ is the dielectric constant, $a = \epsilon/4m_{e-h}e^2$ is the two-dimensional excitonic Bohr radius in the limit $D \to 0$; e is the electron charge, and $m_{e-h} = m_e m_h/(m_e + m_h)$), then the function $Q(\mathbf{p},\omega)$ in the coherent potential approximation is given by[35]

$$Q(\mathbf{p},\omega) = \frac{1}{2}\int G^{(0)}(\mathbf{q},\omega)B(|\mathbf{p}-\mathbf{q}|)\frac{d^dq}{(2\pi)^d}, \tag{3}$$

where

$$B(\mathbf{p}) \equiv \int d^dr B(\mathbf{R})e^{-i\mathbf{pR}}, \tag{4}$$

with $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$; in the coordinate domain $B(\mathbf{R}_1,\mathbf{R}_2)$ has the form[35] (it will be shown below that $B(\mathbf{R}_1,\mathbf{R}_2)$ depends only on $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$)

$$\begin{aligned} B(\mathbf{R}_1,\mathbf{R}_2) &= g\int d^dr \\ &\times\left\{\alpha_e\left(\frac{M}{m_h}\right)^d\left|\varphi_0\left((\mathbf{r}-\mathbf{R}_1)\frac{M}{m_h}\right)\right|^2 - \alpha_h\left(\frac{M}{m_e}\right)^d\left|\varphi_0\left((\mathbf{R}_1-\mathbf{r})\frac{M}{m_e}\right)\right|^2\right\} \\ &\times\left\{\alpha_e\left(\frac{M}{m_h}\right)^d\left|\varphi_0\left((\mathbf{r}-\mathbf{R}_2)\frac{M}{m_h}\right)\right|^2 - \alpha_h\left(\frac{M}{m_e}\right)^d\left|\varphi_0\left((\mathbf{R}_2-\mathbf{r})\frac{M}{m_e}\right)\right|^2\right\}, \end{aligned} \tag{5}$$

where $\varphi_0(r)$ is the ground-state wave function of an exciton, corresponding to the relative motion of the electron and hole ($\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$). For a two-dimensional indirect exciton with spatially separated electron and hole at large interwell distances D ($D \gg a$) the ground-state wave function is given by[40]

$$\varphi_0(r) = \frac{1}{\pi\rho^2(D)}\exp\left(-\frac{r^2}{2\rho^2(D)}\right). \tag{6}$$

Substituting $\varphi_0(r)$ from Eq. (6) into Eq. (5), and assuming $d = 2$, we obtain

$$\begin{aligned} B(R) &= \frac{2(\alpha_e-\alpha_h)^2 g}{\pi^3\rho^2}\exp\left(-\frac{2R^2}{\rho^2}\right), \qquad m_e = m_h; \\ B(R) &= \frac{\alpha_e^2 g}{2\pi^3\rho^2}\exp\left(-\frac{R^2}{\rho^2}\right), \qquad m_h \gg m_e; \quad \alpha_h\left(\frac{m_h}{m_e}\right)^2 \ll \alpha_e. \end{aligned} \tag{7}$$

Using Eq. (4), we obtain the Fourier transform of $B(R)$

$$B(p) = \frac{(\alpha_e - \alpha_h)^2 g}{4\pi^4} \exp\left(-\frac{\rho^2 p^2}{8}\right), \qquad m_e = m_h;$$
$$B(p) = \frac{\alpha_e^2 g}{16\pi^4} \exp\left(-\frac{\rho^2 p^2}{16}\right), \qquad m_h \gg m_e; \quad \alpha_h \left(\frac{m_h}{m_e}\right)^2 \ll \alpha_e. \tag{8}$$

Thus the CPA Green's function of the 2D indirect exciton is determined by the solution of the self-consistent equations Eqs. (2) and (3). In the weak-scattering limit ($g \ll e^2/\epsilon DM$) we use the second-order Born approximation for Q similar to Refs. [35,41], expanding Q (Eq. (3)) in a Taylor series to the first order in $B(|\mathbf{p}-\mathbf{q}|)$ (which is the first order in g), and we replace Eq. (3) by:

$$Q(\mathbf{p},\omega) = \frac{\pi}{2} \int \delta\left(\omega - \frac{q^2}{2M}\right) B(|\mathbf{p}-\mathbf{q}|) \frac{d^d q}{(2\pi)^d}. \tag{9}$$

Substituting $B(p)$ from Eq. (8) into Eq. (9), we obtain for $Q(\mathbf{p},\omega)$

$$Q(\mathbf{p},\omega) = \frac{(\alpha_e - \alpha_h)^2 M g}{16\pi^4} \exp\left(-\frac{\rho^2}{8}(p^2 + 2M\omega)\right) J_0\left(\frac{\rho^2}{4}\sqrt{2M\omega}p\right), \qquad m_e = m_h;$$
$$Q(\mathbf{p},\omega) = \frac{\alpha_e^2 M g}{64\pi^4} \exp\left(-\frac{\rho^2}{16}(p^2 + 2M\omega)\right) J_0\left(\frac{\rho^2}{8}\sqrt{2M\omega}p\right),$$
$$m_h \gg m_e; \quad \alpha_h \left(\frac{m_h}{m_e}\right)^2 \ll \alpha_e, \tag{10}$$

where $J_0(z)$ is a Bessel function of the first kind. The second-order Born Green's function of the single indirect exciton in the random field, $G^{(0)}(\mathbf{p},\omega)$, is derived by substituting $Q(\mathbf{p},\omega)$ from Eq. (10) into Eq. (2).

III. THE SPECTRUM OF COLLECTIVE EXCITATIONS

At small densities n ($n\rho^2 \ll 1$), the system of indirect excitons at low temperatures is a two-dimensional weakly nonideal Bose gas with dipole moments $\mathbf{d}$ normal to wells ($d \sim eD$). The distinction between excitons and bosons manifests itself in exchange effects (see, e.g., Refs.[12],[14], and[42,43]). These effects are suppressed for excitons with spatially separated e and h in a dilute system ($n\rho^2 \ll 1$) at large D ($D \gg a$), because at large D, the exchange interaction in the spatially separated system is suppressed relative to the $e-h$ system in a single well due to the smallness of the tunneling exponent $T \sim \exp[-(D/2a)^{1/4}]$ connected with the penetration of the dipole-dipole interaction through the barrier between the two

wells[12]. Hence, when $D \gg a$, exchange phenomena, connected with the distinction between excitons and bosons, can be neglected, and therefore, the system of indirect excitons in CQWs can be treated by diagram techniques employed for boson systems[38]. Two indirect excitons in a dilute system interact as $U(R) = e^2D^2/(\epsilon R^3)$, where R is the distance between exciton dipoles along quantum well planes.

As mentioned above, in the two-dimensional case the contribution of the dipole interactions can be represented by the sum of the ladder diagrams given in Fig.1. The Bogoliubov approximation for a two-dimensional weakly interacting Bose gas is not valid due to the divergence of the two-dimensional scattering amplitude in the Born approximation[37]. Therefore, for $n\rho^2 \ll 1$, we take into account the direct dipole-dipole repulsion between excitons within the framework of two-dimensional Bose gas theory in the ladder approximation[37]. Since the characteristic frequencies and momenta which give the greatest contribution to the single exciton Green's function $G^{(0)}(\mathbf{p},\omega)$ in the ladder approximation are[37] $\omega\epsilon D/e^2 \sim n/\log[a^2/(nD^4)] \ll 1$ and $p\rho(D) \sim M\sqrt{n/\log[a^2/(nD^4))]} \ll 1$ (at $n\sqrt{aD^3} \ll 1$ and $D \gg a$), respectively, we approximate $Q(\mathbf{p},\omega)$ by $Q(\mathbf{p}=\mathbf{0},\omega=0)$ (see Eq. (10))

$$\begin{aligned} Q(\mathbf{p},\omega) &= Q = \frac{(\alpha_e-\alpha_h)^2 Mg}{16\pi^4}, \qquad m_e = m_h; \\ Q(\mathbf{p},\omega) &= Q = \frac{\alpha_e^2 Mg}{64\pi^4}, \qquad m_h \gg m_e; \quad \alpha_h\left(\frac{m_h}{m_e}\right)^2 \ll \alpha_e. \end{aligned} \tag{11}$$

Note, that the replacing $Q(\mathbf{p},\omega)$ by constant $Q(\mathbf{p}=\mathbf{0},\omega=0)$ does not constitute another additional approximation, but follows from Eq. (10) at small frequencies and momenta, which is applicable for the dilute 2D dipole gas as mentioned above. We show below that the constant Q actually does not affect the ladder approximation vertex $\Gamma(\mathbf{p},\mathbf{p}',0)$ for the weakly interacting Bose gas[38].

We pursue the 2D ladder approximation for the two-particle vertex $\Gamma(p,p';P)$ (Fig.1) analogously to Ref. [37]

$$\Gamma(p,p';P) = U(\mathbf{p}-\mathbf{p}') + s\int \frac{d^3q}{(2\pi)^3} U(\mathbf{p}-\mathbf{q}) G^{(0)}\left(\frac{P}{2}+q\right) G^{(0)}\left(\frac{P}{2}-q\right)\Gamma(q,p';P), \tag{12}$$

where the arguments of the vertex and the Green's function are 3D momentum-frequency vectors (e.g., $p = \{\mathbf{p},\omega\}$), and s is the level degeneracy (equal to 4 for excitons in GaAs quantum wells). Using the single exciton Green's function $G^{(0)}(\mathbf{p},\omega)$ determined by Eq. (2)

with $Q(\mathbf{p},\omega)=Q$ from Eq. (11), we obtain the integral equation for Γ:

$$\Gamma(\mathbf{p},\mathbf{p}';P) \;=\; U(\mathbf{p}-\mathbf{p}')+s\int\frac{d^2q}{(2\pi)^2}\frac{U(\mathbf{p}-\mathbf{q})\Gamma(\mathbf{q},\mathbf{p}';P)}{\frac{\kappa^2}{M}+\Omega-\frac{\mathbf{P}^2}{4M}-\frac{q^2}{M}+2iQ}$$

$$\mu \;=\; \frac{\kappa^2}{2M}=n\Gamma_0=n\Gamma(0,0;0), \tag{13}$$

where $P=\{\mathbf{P},\Omega\}$, and μ is the chemical potential of the system.

The integral equation Eq. (13) for the vertex can be solved analytically in the approximation $\kappa\ll\sqrt{n}$.[37] This inequality must be fulfilled simultaneously with the condition of low density $n\rho^2(D)\ll 1$ (at $n\sqrt{aD^3}\ll 1$ and $D\gg a$) which is necessary for the applicability of the ladder approximation. The solution of the integral equation for the vertex Γ of this system can be expressed through the solution of the equation for the scattering amplitude $f_0(\kappa)\equiv f(\kappa,\kappa)$ of an isolated pair of interacting particles in a two-dimensional system with the repulsive potential $U(R)=e^2D^2/(\epsilon R^3)$:

$$f_0(\kappa)=\frac{\left(\frac{\pi i}{2\kappa}\right)^{1/2}}{\log\left(\frac{\kappa M e^2D^2}{\epsilon}\right)}. \tag{14}$$

The relation of the vertex $\Gamma(\mathbf{p},\mathbf{p}',0)$ (Fig.1.), which represents the effective interaction, to the two-dimensional scattering amplitude $f_0(\mathbf{p}',\mathbf{p})$ is

$$\Gamma(\mathbf{p},\mathbf{p}';0) \;=\; \left[-f_0(\mathbf{p}',\mathbf{p})\frac{2}{M}\left(\frac{2\pi p'}{i}\right)^{1/2}\right]+s\int\frac{d^2K}{(2\pi)^2}\left[-f_0(\mathbf{K},\mathbf{p})\frac{2}{M}\left(\frac{2\pi K}{i}\right)^{1/2}\right]\Gamma(\mathbf{K},\mathbf{p}';0)$$

$$\times\;\left\{\frac{1}{\frac{\kappa^2}{M}-\frac{K^2}{M}+2iQ}-\frac{1}{\frac{p'^2}{M}-\frac{K^2}{M}+i\delta}\right\}, \tag{15}$$

where $\delta\to 0$. In the first order in the scattering amplitude f_0 of the weakly interacting Bose-gas we neglect the second term in the r.h.s. of Eq. (15). Therefore, the ladder approximation vertex Γ does not depend on the constant Q for the weakly interacting Bose gas to first order in the scattering amplitude. Hence, the vertex of the dipole repulsion for the rare exciton gas in the random field will be the same as for the clean system without random field.

Here the characteristic momentum κ, unlike in the three-dimensional system, is not equal to zero but rather is determined from the relation[37]

$$\kappa^2=-4nf_0(\kappa)\left(\frac{2\pi\kappa}{i}\right)^{1/2}. \tag{16}$$

This is a specific feature of two-dimensional Bose system connected with the logarithmic divergence of the two-dimensional scattering amplitude at zero energy. A simple analytical

solution for the chemical potential can be obtained if $\kappa M e^2 D^2/\epsilon \ll 1$. In this limit, the chemical potential μ takes the form:

$$\mu = \frac{\kappa^2}{2M} = \frac{8\pi n}{2M \log\left(\frac{\epsilon^2}{8\pi s^2 n M^2 e^4 D^4}\right)}. \tag{17}$$

Since the only difference between the Green's function of the isolated exciton with and without the random field is the term iQ in Eq. (2), all internal blocks of the ladder approximation diagrams for the system in the random field give the same self-energy as without the random field. This can be shown by repeating the procedure of the derivation of the 2D ladder approximation self-energy for the "clean" system[37] by using integration measure $d(\omega + iQ)$ instead of $d\omega$. The difference in the Green's function will appear only in the external lines as the replacing of $d\omega$ by $d(\omega + iQ)$. Introducing the Green's function of the Bose condensate and normal and anomalous Green's functions of the noncondensate analogously to Ref. [36], we use the finite temperature Green's functions, replacing ω in the zero temperature Green functions by $i\omega_k$, where $\omega_k = 2\pi kT$ (k is an integer[38]; we set the Boltzmann constant $k_B = 1$). This procedure is valid because at small temperatures ($T \ll \mu$) we assume the collective spectrum to be the same as the zero-temperature spectrum[44,45,46]. For the 2D system the temperatures where superfluidity exists can be assumed small, because they are required to be below the Kosterlitz-Thouless temperature T_c[39] (we show below that $T_c < \mu$).

We therefore have the condensate Green's function $D(\mathbf{p}, i\omega_k)$

$$D^{(0)}(\mathbf{p}, i\omega_k) = \frac{-i(2\pi)^2 n_0 \delta(\mathbf{p})}{i\omega_k + \mu + iQ}, \tag{18}$$

where n_0 is the density of Bose condensate. Since at small temperatures $(n - n_0)/n \ll 1$, according to the ladder approximation[38] we use n below instead of n_0. $G(\mathbf{p}, i\omega_k)$ and $F(\mathbf{p}, i\omega_k)$ are the normal and anomalous Green functions of the overcondensate:

$$\begin{aligned} G(\mathbf{p}, i\omega_k) &= -\frac{i\omega_k + \varepsilon_0(p) + \mu + iQ}{\omega_k^2 + \varepsilon^2(p) - 2i(\mu - \varepsilon_0(p))Q}; \\ F(\mathbf{p}, i\omega_k) &= -\frac{\mu}{\omega_k^2 + \varepsilon^2(p) - 2i(\mu - \varepsilon_0(p))Q}, \end{aligned} \tag{19}$$

where $\varepsilon_0(p)$ is the spectrum of noninteracting excitons; the spectrum of interacting excitons has the form $\varepsilon(p) = \sqrt{\left(p^2/(2M) + \sqrt{\mu^2 - Q^2}\right)^2 - (\mu^2 - Q^2)}$, and for small momenta $p \ll \mu$ the excitation spectrum is acoustic $\varepsilon(p) = c_s p$, where $c_s = \sqrt{\sqrt{\mu^2 - Q^2}/M}$ is the velocity of sound.

From the expression for the spectrum $\varepsilon(p)$ we see that for $Q > \mu$ the Bose condensate state becomes unstable, because the spectrum $\varepsilon(p)$ becomes imaginary. So, for $Q > \mu$ the random field destroys superfluidity, even at $T = 0$. This condition for the instability of the condensate itself is very approximate, because we have used the ladder approximation, which is valid only if almost all paricles are in the condensate ($(n - n_0)/n \ll 1$), which is not the case for large random field contribution Q. Since at $Q < \mu$ the spectrum of the system is acoustic and satisfies the Landau criterium of superfluidity, the system becomes a two-component liquid, consisting of the superfluid and normal component in the presence of the random field even at $T = 0$[38]. As Q grows, the system undergoes a transition to the exciton glass state (for the a lattice model of bosons, this transition was considered in Ref.[47]).

IV. THE KOSTERLITZ-THOULESS PHASE TRANSITION

The density of the superfluid component $n_s(T)$ can be obtained using the relation $n_s(T) = n - n_n(T)$, where $n_n(T)$ is the density of the normal component.

The density of the normal component $n_n(T)$, which is dissipated at the walls and impurities, can be calculated using the Kubo formula as the response of the total momentum to an external velocity[48]:

$$n_n = -\lim_{\omega \to 0} \left[\frac{Im(\Pi(i\omega))}{i\omega} \right], \tag{20}$$

where $\Pi(i\omega)$ is the polarization operator with zero transferred momentum

$$\Pi(i\omega) = \frac{1}{2M} s \sum_{\mathbf{p}} p^2 T \sum_{\omega'_k} \mathcal{F}(\mathbf{p}, i\omega'_k + i\omega)\mathcal{F}(\mathbf{p}, i\omega'_k), \tag{21}$$

and $\mathcal{F}(\mathbf{p}, i\omega_k)$ is the total single-particle Matsubara Green's function of an indirect exciton

$$\mathcal{F}(\mathbf{p}, i\omega'_k) = D(\mathbf{p}, i\omega'_k) + G(\mathbf{p}, i\omega'_k). \tag{22}$$

The renormalization of the vertex by the interaction is neglected in the polarization operator Eq. (21). When the interaction is taken into account in the ladder approximation, a term which is small with respect to the parameter $M\Gamma \ll 1$ appears (Γ is the vertex in the ladder approximation). For a two-dimensional rarefied system of indirect excitons this parameter has the form $4\pi / \log\left[(\epsilon^2/(8\pi s^2 n M^2 e^4 D^4)\right] \ll 1$.

We now substitute the Green's functions of the condensate Eq. (18) and noncondensate Eq. (19) particles into Eq. (22). Next, substituting the expression Eq. (22) into Eqs. (21) and (20) we have

$$n_n = n_n^0 + s\frac{N}{M}\int \frac{d\mathbf{p}}{(2\pi)^2} p^2 \mu \frac{\varepsilon_0(p)}{\varepsilon^4(p)} Q. \tag{23}$$

Here N is the total number of particles, and n_n^0 is the density of the normal component in a pure system with no impurities:

$$n_n^0 = -s\frac{1}{2M}\int \frac{d\mathbf{p}}{(2\pi)^2} p^2 \frac{\partial n_0(p)}{\partial \varepsilon}. \tag{24}$$

where $n_0(p) = (e^{\varepsilon(p)/T} - 1)^{-1}$ is the distribution of an ideal Bose gas of temperature excitations.

The first term in Eq. (23), which does not depend on Q, is the contribution to the normal component due to scattering of quasiparticles with an acoustic spectrum in an ordered system at $T \neq 0$. In a two-dimensional system,

$$n_n^0 = s\frac{3\zeta(3)}{2\pi}\frac{T^3}{c_s^4(n,Q)M}, \tag{25}$$

where $\zeta(z)$ is the Riemann zeta function ($\zeta(3) \simeq 1.202$). The second term in Eq. (23) is the contribution to the normal component due to the interaction of the particles (excitons) with the random field,

$$n_n = n_n^0 + s\frac{nQ}{2Mc_s^2(n,Q)} = s\frac{3\zeta(3)}{2\pi}\frac{T^3}{c_s^4(n,Q)M} + s\frac{nQ}{2Mc_s^2(n,Q)}. \tag{26}$$

The density of the superfluid component is $n_s = n - n_n$. From Eqs. (25) and (26) we can see that the random field decreases the density of the superfluid component.

In a 2D system, superfluidity appears below the Kosterlitz-Thouless transition temperature $T_c = \pi n_s/(2M)$,[39] where only coupled vortices are present. Using the expressions (25) and (26) for the density n_s of the superfluid component, we obtain an equation for the Kosterlitz-Thouless transition temperature T_c. Its solution is

$$T_c = \left[\left(1 + \sqrt{\frac{32}{27}\left(\frac{MT_c^0}{\pi n'}\right)^3 + 1}\right)^{1/3} - \left(\sqrt{\frac{32}{27}\left(\frac{MT_c^0}{\pi n'}\right)^3 + 1} - 1\right)^{1/3}\right]\frac{T_c^0}{2^{1/3}}. \tag{27}$$

Here T_c^0 is an auxiliary quantity, equal to the temperature at which the superfluid density vanishes in the mean-field approximation (i.e., $n_s(T_c^0) = 0$),

$$T_c^0 = \left(\frac{2\pi n' c_s^4 M}{3\zeta(3)}\right)^{1/3}. \tag{28}$$

In Eqs. (27) and (28), n' is

$$n' = n - s\frac{nQ}{2Mc_s^2}. \tag{29}$$

V. DISCUSSION

The dependence of the Kosterlitz-Thouless transition temperature T_c as a function of the total exciton density n for different Q, obtained from Eq. (27), is presented in Fig. 2. It can be seen in Fig. 2 that the random field decreases the Kosterlitz-Thouless transition temperature. This trend was pointed out before, for weak coupling with the random field, in Ref. [36]. Figs. 3 and 4 show the dependence of the Kosterlitz-Thouless transition temperature on the random field parameter Q and the spatial separation between the electrons and holes.

The results of the approximation used in Ref. [36] can be obtained from the normal density n_n, derived in the present work (Eq. (26)), as a first order in the expansion of n_n in Eq. (26) respect to Q/μ, which corresponds to the case when the random field is weaker than the dipole-dipole repulsion and $Q/\mu \ll 1$. (Operationally, this corresponds to replacing the speed of sound by its value at $Q = 0$, i.e., $c_s \to \sqrt{\mu/M}$, in Eq. (26).) For realistic experimental parameters, the random field is not always smaller than the dipole-dipole repulsion. (E.g., in Ref. [49], the luminescence linewdith due to inhomogeneous broadening at low density is approximately 2 meV; more recent GaAs structures[16] have inhomogeneously broadened linewidths closer to 1 meV.) Figs. 5 and 6 show that the approach used in the present work results in the Kosterlitz-Thouless temperature being smaller than the transition temperature obtained from the approximation used in Ref. [36], which is denoted here by PT (for perturbation theory; see below). The difference in the Kosterlitz-Thouless temperature between these two approaches increases when random field Q increases and exciton density n decreases (Figs. 5 and 6). The results of this comparison are reasonable, because the approximation used in Ref. [36] implies a first-order perturbation theory with respect to Q/μ for the Green's function, and the inequality $Q/\mu \ll 1$ holds for small Q and not very small densities n (μ increases as n increases). Note that in the present work the parameter Q/μ is not required to be small.

This work shows that although the random field depletes the condensate, Kosterlitz-Thouless superfluidity is still possible in a system of spatially indirect excitons. However, for realistic experimental parameters, the possibility of superfluidity is marginal. For the

structure parameters used in the plots of Figs. 2-4, with effective $D = 15\ nm$, the binding energy of the indirect excitons at high electric field is 3.5 meV and the excitonic Bohr radius is around 150 Å. This implies that $na^2 \cong 1$, the point at which the exciton gas becomes plasma-like, at a density of around 5×10^{11} cm^{-2}. If the exciton density is to be well below this value, say an order of magnitude, then as seen in Fig. 2, the disorder parameter Q must be around 0.1-0.2 meV. By comparison, the half-width, half maximum of the exciton luminescence line at low temperature and low density in recent samples, due to inhomogeneous broadening, is around 0.5 meV[16]. It is not clear exactly how the inhomogeneous line width and the disorder factor Q are related. The line width is likely sensitive to disorder with length scales short compared to the excitonic Bohr radius, while the Q parameter used here is assumed to be a measure of disorder on length scales long compared to the excitonic Bohr radius. Nevertheless, there is still a problem with Kosterlitz-Thouless superfluidity at low density, no matter what the temperature.

The story may change when the excitons are trapped in a confining potential, in which case true Bose condensation is possible in the Stringari-Pitaevskii limit.[29] However, these results indicate that the role of disorder must be taken into account for any realistic treatment of excitonic condensation.

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Captures to Figures (1-6)

Fig.1. Diagrammatic representation of the equation for the vertex Γ in the momenta-frequency representation $(\mathbf{P}, \Omega)$.

Fig.2. Dependence of temperature of Kosterlitz-Thouless transition $T_c = T_c(n)$ (in units of K; for GaAs/AlGaAs: $M = 0.24m_0$, where m_0 is the free electron mass; $\epsilon = 13$) on the exciton density n (in units of cm^{-2}) at the interwell distance $D = 15\ nm$, for different random fields Q (in units of meV): $Q = 0$ – solid curve (straight line); $Q = 0.1\ meV$ – dotted curve; $Q = 0.2\ meV$ – dashed curve; $Q = 0.3\ meV$ – long-dashed curve; $Q = 0.4\ meV$ – dashed-dotted curve; $Q = 0.5\ meV$ – solid curve (bottom right hand corner).

Fig.3. Dependence of temperature of Kosterlitz-Thouless transition $T_c = T_c(Q)$ (in units of K; for GaAs/AlGaAs: $M = 0.24m_0$; $\epsilon = 13$) on the random field Q (in units of meV) at the interwell distance $D = 15\ nm$, for different exciton densities n: $n = 5 \times 10^{10}\ cm^{-2}$ – solid curve; $n = 1.0 \times 10^{11}\ cm^{-2}$ – dotted curve; $n = 3.0 \times 10^{11}\ cm^{-2}$ – dashed curve.

Fig.4. Dependence of temperature of Kosterlitz-Thouless transition $T_c = T_c(D)$ (in units of K; for GaAs/AlGaAs: $M = 0.24m_0$; $\epsilon = 13$) on the interwell distance D (in units of nm) at the exciton density $n = 1 \times 10^{11}\ cm^{-2}$, for different random fields Q: $Q = 0$ – solid curve; $Q = 0.3\ meV$ – dotted curve; $Q = 0.5\ meV$ – dashed curve; $Q = 0.6\ meV$ – dashed-dotted curve.

Fig.5. Temperature dependence of Kosterlitz-Thouless transition $T_c = T_c(n)$ based on CPA and the perturbation theory (PT) approximation used in Ref. [36] (in units of K; for GaAs/AlGaAs: $M = 0.24m_0$; $\epsilon = 13$; m_0 is a mass of electron) on the exciton density n (in units cm^{-2}) at the interwell distance $D = 15\ nm$, for different random fields Q (in units of meV): $Q = 0.2\ meV$ – solid curve, full CPA *vs.* dotted curve, PT; $Q = 0.3\ meV$ – dashed curve, full CPA *vs.* long-dashed curve, PT.

Fig.6. Temperature dependence of Kosterlitz-Thouless transition $T_c = T_c(Q)$ based on CPA and the perturbation theory (PT) approximation used in Ref. [36] (in units of K; for GaAs/AlGaAs: $M = 0.24m_0$; $\epsilon = 13$) on the random field Q (in units of meV) at the interwell distance $D = 15\ nm$, for different exciton densities n: $n = 5 \times 10^{10}\ cm^{-2}$ – solid curve, full CPA *vs.* dotted curve, PT; $n = 1 \times 10^{11}\ cm^{-2}$ – dashed curve, full CPA *vs.* long-dashed curve, PT.

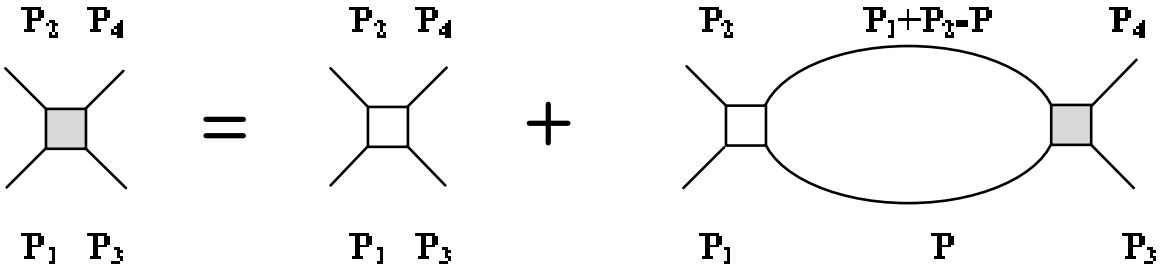

Fig.1.

Figure 2

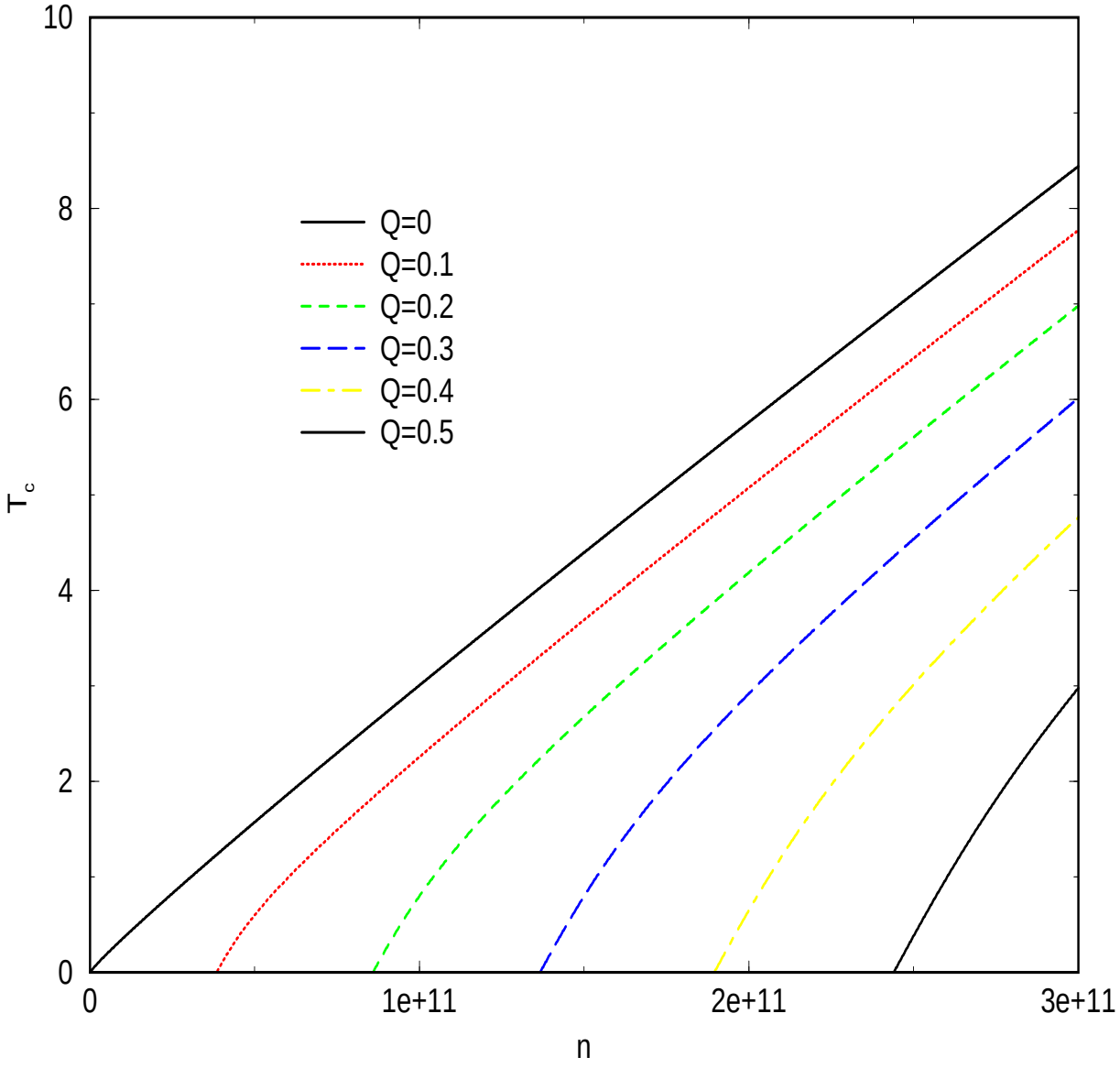

Figure 3

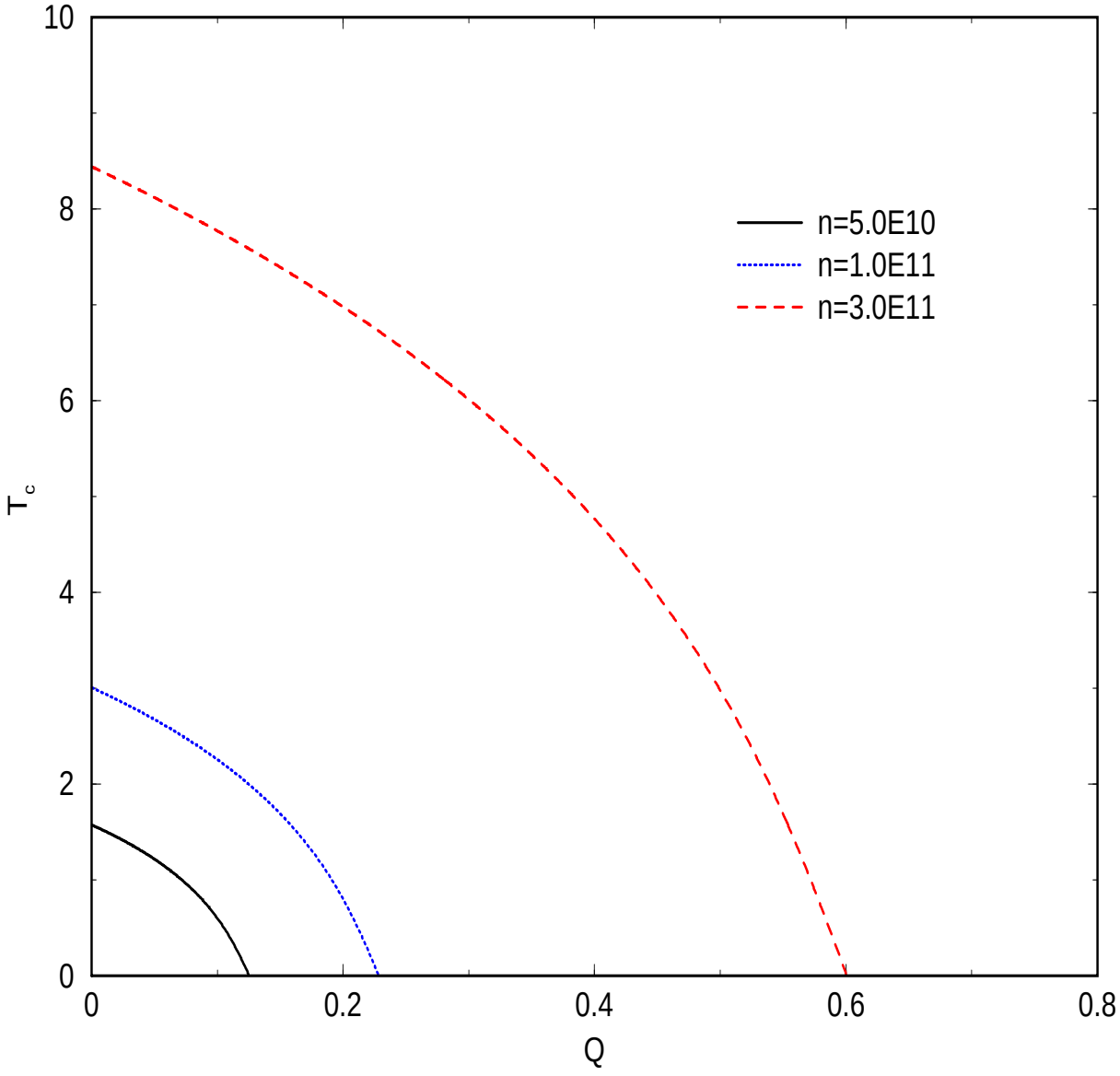

Figure 4

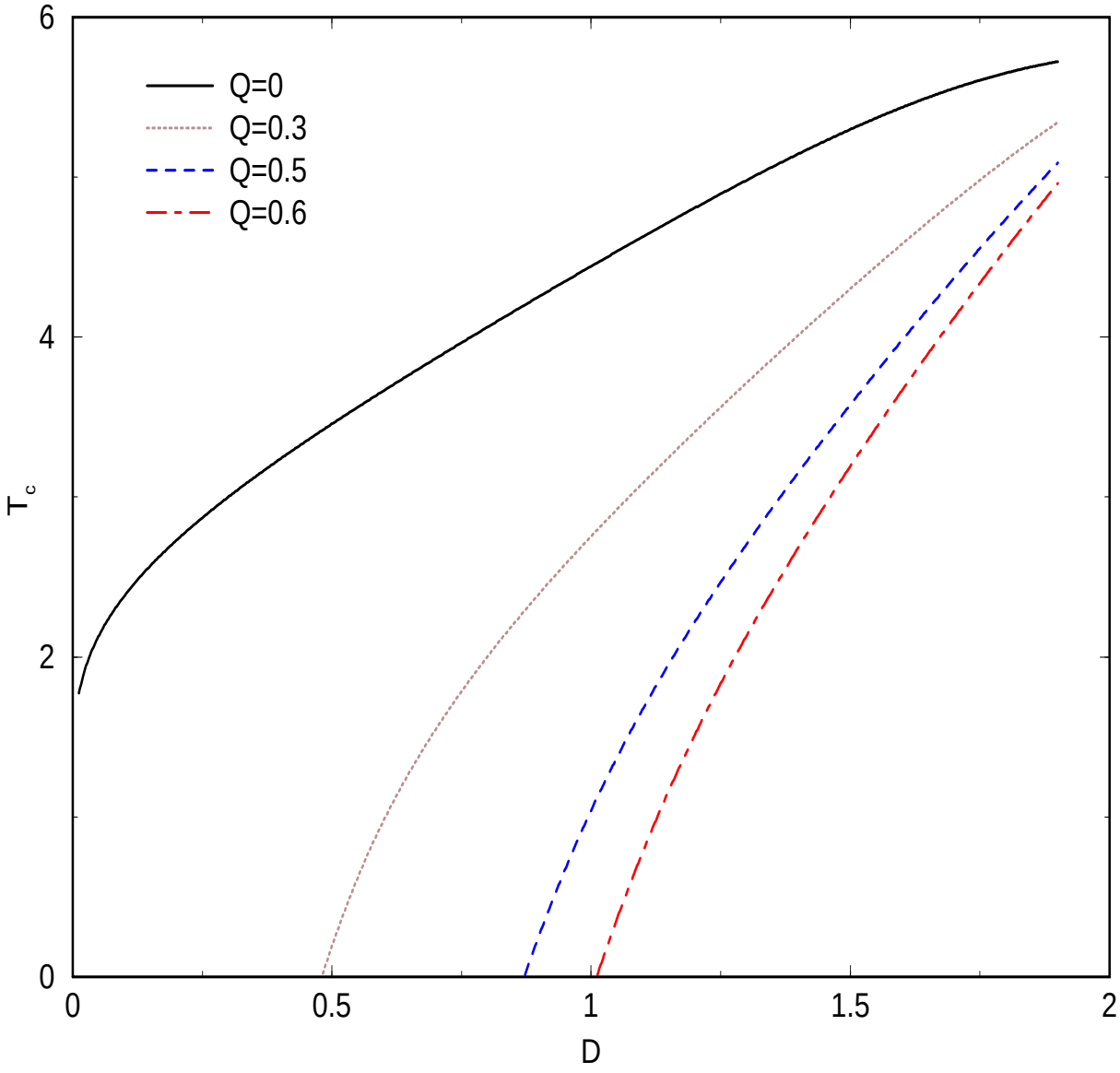

Figure 5

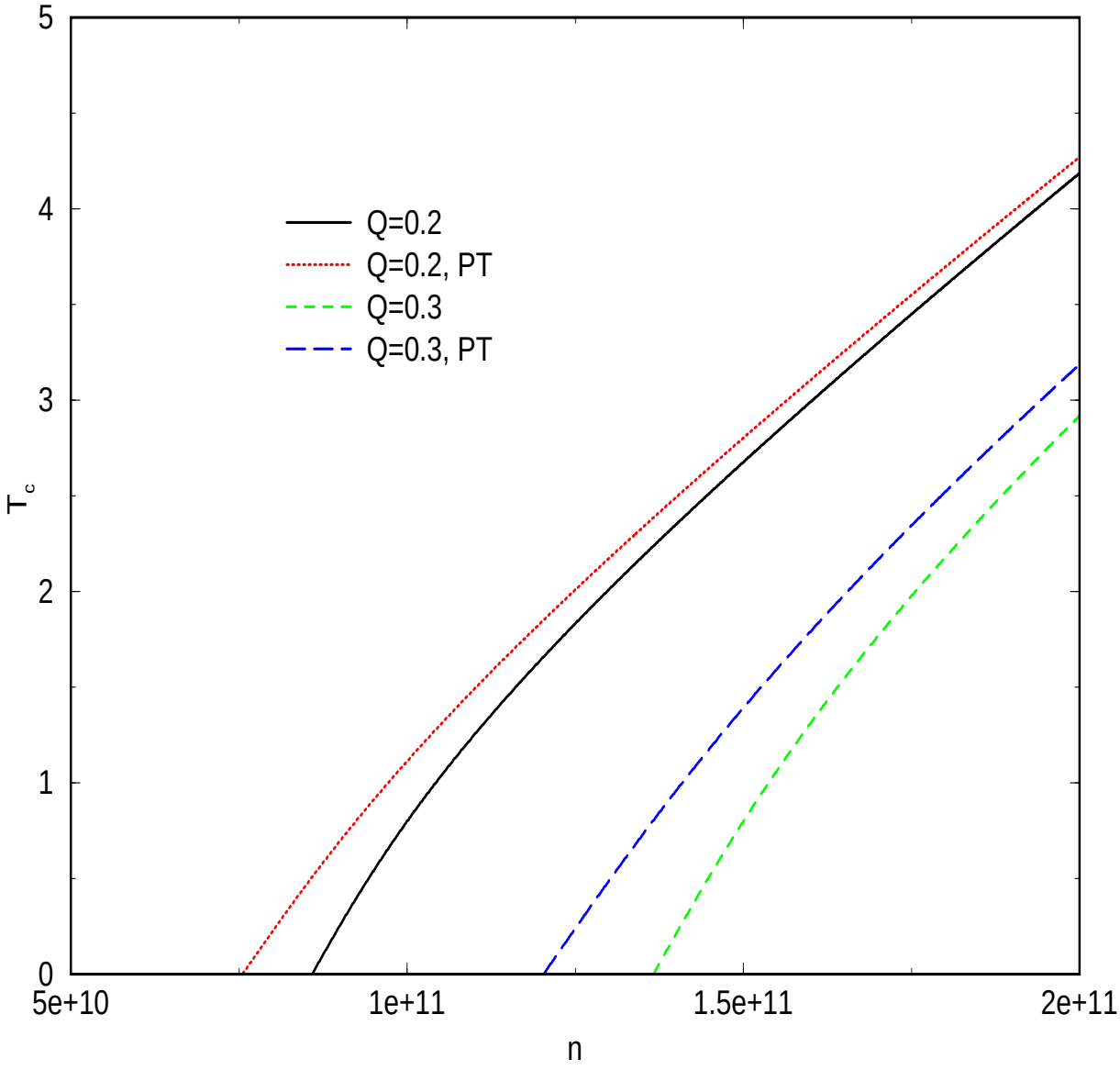

Figure 6